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Pseudoscalar meson masses in unitarized chiral perturbation theory

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Abstract. This contribution summarizes the work explained in arXiv:hep-ph/0608290 where we perform a non-perturbative chiral study of the masses of the lightest pseudoscalar mesons. The pseudoscalar self-energies are calculated by the evaluation of the scalar self-energy loops with full S-wave meson-meson amplitudes taken from Unitary Chiral Perturbation Theory (UCHPT). These amplitudes, among other features, contain the lightest nonet of scalar resonances σ, $f_0(980)$, $a_0(980)$ and κ. The self-energy loops are regularized by a proper subtraction of the infinities within a dispersion relation formulation of the amplitudes. Values for the bare masses of pions and kaons and the η_8 mass are obtained. We then match to the self-energies from standard Chiral Perturbation Theory (CHPT) to $\mathcal{O}(p^4)$ and resum higher orders from our calculated scalar self-energies. The dependence of the self-energies on the quark masses allows a determination of the ratio of the strange-quark mass over the mean of the lightest-quark masses, $m_s/\hat{m}$, in terms of the $\mathcal{O}(p^4)$ CHPT low-energy constant combinations $2L_8^r - L_5^r$ and $2L_6^r - L_4^r$. In this way, we give a range for the values of these low-energy counterterms and for $3L_7 + L_8^r$, once the η-meson mass is invoked. The low-energy constants are further constraint by performing a fit to the recent MILC lattice data on the pseudoscalar masses, and $m_s/\hat{m} = 25.6 \pm 2.5$ results. This value is consistent with 24.4 ± 1.5 from CHPT and phenomenology and more marginally with the value 27.4 ± 0.5 obtained from pure perturbative chiral extrapolations of the MILC lattice data to physical values of the lightest-quark masses.

PACS. 14.65.Bt Light quarks – 12.40.Yx Hadron mass models and calculations – 14.40.Aq π, K, and η mesons – 12.39.Fe Chiral Lagrangians

Three-flavour CHPT has already reached a very sophisticated stage with present calculations at the level of next-to-next-to-leading order (NNLO). In ref. [1] the masses and decay constants of the lightest octet of pseudoscalar mesons are worked out at NNLO. One striking fact concerns the much larger $\mathcal{O}(p^6)$ two-loop contribution than the full $\mathcal{O}(p^4)$ one to the self-energies of the pseudoscalars, by around one order of magnitude. As stated in ref. [1] the $\mathcal{O}(p^6)$ counterterms "seem severely overestimated" due to the complicated nature of the scalar sector. Hence, this study rises the interesting question of how one can improve the calculation of the contributions from the scalar sector. This is an object of study of the investigation done in ref. [2], which is the work we summarize in the present contribution. Concerning the importance of the S-wave dynamics, it is well known that the scalar sector is the source of large higher-order corrections to the CHPT series, because of the enhancement of unitarity diagrams, which definitely slow down the convergence of the perturbative chiral series. A novel example, where these large corrections play a role, as we show in this work, concerns the determinations of the light-quark masses from present lattice calculations of the pseudoscalar masses [3, 4] with three dynamical fermions. In these evaluations one confronts with the problematic S-wave dynamics plagued of non-perturbative effects. The MILC Collaboration reports a value for the ratio $m_s/\hat{m} = 27.4 \pm 0.5$, with $\hat{m} = (m_u + m_d)/2$, inconsistent with its determination from CHPT and phenomenology [5], $m_s/\hat{m} = 24.4 \pm 1.5$. Because Unitary CHPT (UCHPT) has proved very successful in dealing, precisely, with the strong meson-meson S-wave interactions, it is worth employing this scheme as a parameterization to bring down the results from lattice QCD to the physical lightest-quark masses. Here, we will employ $\mathcal{O}(p^4)$ CHPT and supply it with the corrections from UCHPT of $\mathcal{O}(p^6)$ and higher.

In the following, we briefly summarize the formalism and results of ref. [2]. The starting point in ref. [2] is the S-wave meson-meson partial waves both for resonant isospins $(I) = 0$, 1, 1/2 as well as for the much smaller and non-resonant ones, $I = 3/2$ and 2. For the former set we take the amplitudes of ref. [6]. It is worth stressing that this is just a subset of all possible S-wave two-body rescattering diagrams. Nonetheless, these have been

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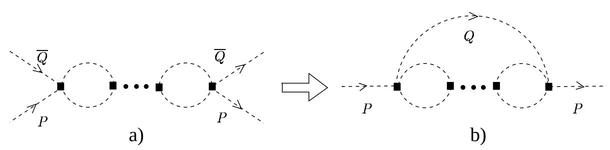

Fig. 1. a) represents the S-wave amplitude $P\overline{Q} \to P\overline{Q}$ and b) is the diagram for the calculation of the self-energy of the pseudoscalar P due to the intermediate pseudoscalar Q.

shown to be the dominant contributions and enough to explain a wide range of phenomenology like, *e.g.*, meson-meson scattering [7,6,8], $\gamma\gamma \to$ meson-meson [9], meson form factors [10–12], ϕ [13], J/Ψ [10,14], η [15], B and D [16] decays, determinations of light-quark masses and V_{us} [17], etc.

The interaction kernel employed in ref. [6] comprises the lowest-order CHPT amplitudes together with the exchange of s-channel scalar resonances in a chiral symmetric invariant way. The low-lying scalar resonances are generated even when no explicit resonances at tree level are included [7,6]. The basic point in UCHPT is to resum the right-hand or unitarity cut to all orders, which is the source of the large corrections produced by the S-wave meson-meson interactions, and perform a chiral expansion of the rest, the so-called interaction kernel, calculating it perturbatively from CHPT [18,19,6].

The set of diagrams enhanced in S-wave meson-meson scattering are represented in fig. 1a, for the process $P\overline{Q} \to P\overline{Q}$. Our expression for the self-energy of the pseudoscalar P due to the loops schematically represented in fig. 1 can be expressed by the sum,

$$\Sigma_P^U = -\sum_Q \int \frac{\mathrm{d}^3 k}{(2\pi)^3 2E_Q(\mathbf{k})} T_{P\overline{Q}\to P\overline{Q}}(s_1), \qquad (1)$$

with $Q = \{\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \overline{K}^0, \eta\}$ and $s_1 = (M_P - E_Q(\mathbf{k}))^2 - \mathbf{k}^2$ for $p = (M_P, \mathbf{0})$. In addition one has to add a tadpole contribution coming from the t-crossed process $P\overline{P} \to Q_i\overline{Q_i}$ which has a similar expression. The loop function of eq. (1) is quadratically divergent. By performing a once subtracted dispersion relation for the scattering amplitudes, it is easy to identify the origin of the infinites and get rid of them up to logarithmic pieces. (See ref. [2] for details.)

The mass equation one has to solve is

$$M_P^2 = \overset{\circ}{M}{}_P^2 + \Sigma_P(\overset{\circ}{M}{}_Q^2; M_Q^2). \qquad (2)$$

The dependence on the bare masses $\overset{\circ}{M}{}_Q^2$ originates from the dependence on the explicit inclusion of the quark mass matrix in CHPT from where the interaction kernel $\mathcal{K}$ is calculated [6]. By solving exactly eq. (2) we find $\overset{\circ}{M}_\pi = 126 \pm 4\,\mathrm{MeV}$, $\overset{\circ}{M}_K = 476 \pm 10\,\mathrm{MeV}$, $M_{\eta_8} = 635 \pm 15\,\mathrm{MeV}$. Regarding the sizes of the self-energies, we have, $\Sigma_P(\overset{\circ}{M}{}_Q^2; M_Q^2)/M_P^2 = 0.11 \pm 0.06$, 0.08 ± 0.04, 0.26 ± 0.06, for pions, kaons and etas, respectively. We observe that most of the physical masses of the pseudoscalars is due to the bare mass. Taking into account the expression for the bare masses in terms of the quark masses and the values of the former ones given above, one get, $r_m = \frac{m_s}{\hat{m}} = 2\frac{\overset{\circ}{M}{}_K^2}{\overset{\circ}{M}{}_\pi^2} - 1 = 27.1 \pm 2.5$.

On the other hand, one can split the self-energy into the $\mathcal{O}(p^4)$ contribution and higher orders:

$$M_P^2 = \overset{\circ}{M}{}_P^2 + \Sigma_P^{4\chi}(\overset{\circ}{M}{}_Q^2; L_i^r) + \Sigma_P^H(\overset{\circ}{M}{}_Q^2; M_Q^2), \qquad (3)$$

where $\Sigma_P^H(\overset{\circ}{M}{}_Q^2; M_Q^2)$ is the self-energy calculated removing explicitely the $\mathcal{O}(p^4)$ contribution and $\Sigma_P^{4\chi}(\overset{\circ}{M}{}_Q^2; L_i^r)$ is the $\mathcal{O}(p^4)$ CHPT contribution from [20]. Equation (3) incorporates the combinations of the low-energy constants $L_{(5,8)} \equiv 2L_8^r - L_5^r$, $L_{(4,6)} \equiv 2L_6^r - L_4^r$ and $L_{(7,8)} \equiv 3L_7 + L_8^r$. As a function of $L_{(4,6)}$ and $L_{(5,8)}$, we solve first for the $\overset{\circ}{M}{}_\pi^2$ and $\overset{\circ}{M}{}_K^2$ bare masses in eqs. (3). Once we calculate $\overset{\circ}{M}{}_\pi^2$ and $\overset{\circ}{M}{}_K^2$, we fix $\overset{\circ}{M}{}_\eta^2$ using the Gell-Mann-Okubo relation and from the η equation, we then solve for $L_{(7,8)}$. In fig. 2 we show by contour plot lines the calculated values for $r_m = m_s/\hat{m}$ as a function of $L_{(4,6)}$ and $L_{(5,8)}$. (All the values of the LECs displayed are given in units of 10^{-3}.) If one takes into account the value of $m_s/\hat{m}$ from CHPT [5], 24.4 ± 1.5, the preferred region of $2L_6^r - L_4^r$ and $2L_8^r - L_5^r$ would be about in between the 23 and 26 contour lines. The shadowed areas below and above these lines represent the uncertainties of our calculation in these lines due to the variation in the renormalization scale, $\mu \sim [0.5, 1.2]\,\mathrm{GeV}$, and in the input parameters for the S-waves. Hence, they delimit the allowed bounded region of $2L_6^r - L_4^r$ and $2L_8^r - L_5^r$ that is permitted if the result $m_s/\hat{m} = 24.4 \pm 1.5$ [5] is taken.

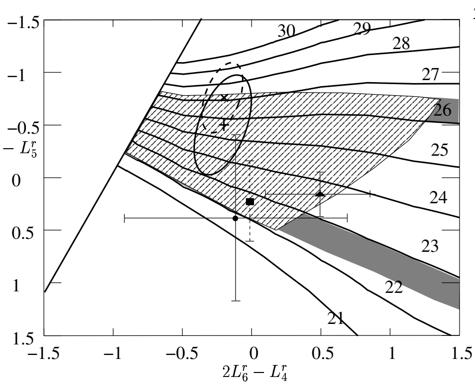

Fig. 2. Contour plot for $r_m = m_s/\hat{m}$ as a function of $L_{(4,6)}$ and $L_{(5,8)}$ showing also our point from the result of the fit to lattice data together with our theoretical uncertainty represented by the solid ellipse. The dashed ellipse represents the value for the LECs, within uncertainty, needed to reproduce the bare masses obtained with the full dynamical model. On the other hand, the triangle point corresponds to the lattice extrapolation from Staggered CHPT [21]. The circle and square points correspond, respectively, to the $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ CHPT results from refs. [22,23].

In order to obtain a sharper determination for the self-energies and quark mass ratio r_m, we employ our eq. (3) to reproduce the quark mass dependence of the recent lattice determinations of pseudoscalar masses from the MILC Collaboration [3] with three dynamical quarks, u, d and s and two lattice spacings $a_{coarse} = 0.12\,\text{fm}$ and $a_{fine} = 0.09\,\text{fm}$. Our aim here in ref. [2] was just to have more points with which to compare, including those for unphysical values of quark masses and not only the physical ones, so that we can obtain a more constraint determination for $L_{(5,8)}$ and $L_{(4,6)}$. The MILC Collaboration [3] provides pseudoscalar meson masses as a function of the quark ones, measured in terms of the lattice spacing a, which is known with an error of about 1.2%. Current quark masses and those of lattice are proportional [4], so we write the bare meson masses in terms of the lattice quark masses as, $\overset{\circ}{M}{}^2_\pi = 2B_0C(a)a\hat{m}$, $\overset{\circ}{M}{}^2_K = B_0C(a)a(\hat{m}+m_s)$, $\overset{\circ}{M}{}^2_\eta = \frac{2}{3}B_0C(a)a(\hat{m}+2m_s)$, and treat $B_0C(a)$ as a free parameter in our fits to the three flavour runs of ref. [3]. We notice that $C(a_{fine}) = 1.49C(a_{coarse})$ [2]. We obtain a very good fit to the lattice data. From the fit we get the values, $2L_8^r - L_5^r = -0.52 \pm 0.43$, $2L_6^r - L_4^r = -0.20 \pm 0.17$. These values can be compared with the result from the MILC Collaboration [21] $2L_8^r - L_5^r = 0.16 \pm 0.2$, $2L_6^r - L_4^r = 0.4 \pm 0.4$, given by the triangle point in fig. 2. The corresponding ratio of quark masses is then,

$$m_s/\hat{m} = 25.6 \pm 2.5. \tag{4}$$

This value is in perfect agreement with 24.4 ± 1.5 from ref. [5]. The previous result is also in agreement, at the level of one sigma, with the value 27.4 ± 0.5 determined in ref. [4]. However, the difference between the central values for r_m obtained and the one of ref. [4] is rather large, 2 units. At this point, it is worth stressing that we have been able to give, with a non-perturbative chiral parameterization, a remarkably good reproduction of the lattice data while giving rise to a value for r_m significantly smaller than that of ref. [4] and in the range of values previously predicted from CHPT and phenomenology [5]. Thus, we certainly can conclude that it is not a *necessary* feature of the lattice runs of pseudoscalar masses by the MILC Collaboration [4] having a significantly larger $m_s/\hat{m}$ value than that previously determined in ref. [5]. With respect to the bare masses one has, $\overset{\circ}{M}_\pi = 125.0 \pm 4.3\,\text{MeV}$, $\overset{\circ}{M}_K = 456.2 \pm 20.8\,\text{MeV}$. Regarding $L_{(7,8)}$ we get $L_{(7,8)} = -0.6 \pm 0.6$, in good agreement with typical results in the literature, although with large errors. The dashed ellipse in fig. 2 represents the value for the LECs needed to reproduce the results for the bare masses in the full dynamical case. From there one can conclude that the dynamical scalar resonance saturation of $L_{(4,6)}$ is very good but poorer for $L_{(5,8)}$, although still compatible within errors.

We also explicitely show in table 1 the relative sizes of $\Sigma_P^{4\chi}$ and Σ_P^H. We also get that the ratio between the $\mathcal{O}(p^4)$ and the $\mathcal{O}(p^6)$ and higher-order contributions is quite sensitive to the LECs considered. The sizes of our self-energies from Σ_P^H are rather similar to those determined in ref. [23] to $\mathcal{O}(p^6)$. Reference [1] obtains: $\Sigma_\pi^{6\chi}/M_\pi^2 = 0.132$–$0.355$, $\Sigma_K^{6\chi}/M_K^2 = 0.194$–$0.423$ and $\Sigma_\eta^{6\chi}/M_\eta^2 = 0.234$–$0.521$. These values are well inside our bulk of results for $\Sigma_\pi^H/M_\pi^2 = 0.222 \pm 0.04$, $\Sigma_K^H/M_K^2 = 0.375 \pm 0.07$, $\Sigma_\eta^H/M_\eta^2 = 0.506 \pm 0.09$. Thus, the calculations of ref. [23] to $\mathcal{O}(p^6)$, although showing that this order is much larger than the $\mathcal{O}(p^4)$, does not imply necessarily the lack of convergence of the chiral series since our calculation, estimating higher-orders corrections by incorporating physical S-waves which include both resonant and non-resonant physics, gives us values of similar size to those of this reference up to $\mathcal{O}(p^6)$.

Table 1. Relative sizes of the $\mathcal{O}(p^4)$, $\mathcal{O}(p^6)$ and higher-order contributions, $\Sigma_P^{4\chi}$ and Σ_P^H, respectively, to the self-energies.

	π	K	η
$\Sigma_P^{4\chi}/M_P^2$	-0.0789 ± 0.060	-0.221 ± 0.087	-0.410 ± 0.193
Σ_P^H/M_P^2	0.222 ± 0.04	0.375 ± 0.07	0.506 ± 0.09
$\Sigma_P^{4\chi}/\Sigma_P^H$	-0.355 ± 0.287	-0.589 ± 0.236	-0.810 ± 0.384

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